Quantum Simulations and Quantum Walks 2014

Pumula Beach Hotel

24-28 November 2014

Programme

Monday, 24 November

Session Chair: Francesco Petruccione

$14^{30} - 15^{00}$ Registration

$15^{00} - 15^{30}$ Opening (Jonathan Blackledge - Deputy Vice-Chancellor Research, University of KwaZulu-Natal)

$15^{30} - 16^{30}$ Stéphane Attal (Université de Lyon), Classical Noise in Quantum Markov chains.

$16^{30} - 17^{00}$ Tea/Coffee

$17^{00} - 17^{30}$ Armando Perez (University of Valencia), Asymptotic properties of the Dirac quantum cellular automaton

$17^{30} - 18^{00}$ Stefano Facchini (Université Grenoble-Alpes), Discrete Lorentz transformations for Quantum Walks and Quantum Cellular Automata

$18^{00} - 19^{00}$ Welcome reception

$19^{00} - 21^{00}$ Dinner

Tuesday, 25 November

Session Chair: Stéphane Attal

$9^{00} - 10^{00}$ Denis Bernard (Ecole Normale Supérieure de Paris), The Open Quantum Brownian Motion: Fluctuation and (possible) Geometrisation.

$10^{00} - 10^{30}$ Ilya Sinayskiy (NITheP and University of KwaZulu-Natal), An example of the microscopic derivation of Open Quantum Brownian Motion.

$10^{30} - 11^{00}$ Tea/Coffee

$11^{00} - 11^{30}$ Francesco Petruccione (NITheP and University of KwaZulu-Natal), Open Quantum Walks

$11^{30} - 12^{00}$ Hazmatally Goolam Hossen (University of KwaZulu-Natal), Open Quantum Self-avoiding Walks

$12^{30} - 14^{30}$ Lunch

Session Chair: Denis Bernard

$15^{00} - 16^{00}$ Dominic Berry (Macquarie University), Hamiltonian simulation with nearly optimal dependence on all parameters

$16^{00} - 16^{30}$ Miguel Navascues (Autonomous University of Barcelona), Noncommutative polynomial optimization and simulations with quantum resources.

$16^{30} - 17^{00}$ Tea/Coffee

$17^{00} - 17^{30}$ Ryan Sweke (University of KwaZulu-Natal), Digital quantum simulation of single-qubit open quantum systems

$17^{30} - 18^{00}$ Anna Przysiezna (University of Gdansk), Lattice shaking for topological effects in cold atomic quantum simulators.

$19^{00} - 21^{00}$ Dinner

Wednesday, 26 November

Session Chair: Dominic Berry

$9^{00} - 10^{00}$ Alessandro Fedrizzi (University of Queensland), Observation of environmentally-assisted quantum transport in a photonic emulator.

$10^{00} - 10^{30}$ Etsuo Segawa (Tohoku University), Quantum walks on infinite trees

$10^{30} - 11^{00}$ Tea/Coffee

$11^{00} - 11^{30}$ Christopher Cedzich (Leibnitz Universität Hannover), Bulk-edge correspondence in one-dimensional quantum walks

$11^{30} - 12^{00}$ Christoph Stahl (Leibnitz Universität Hannover), Quantum Walks in Electromagnetic Fields

$12^{30} - 14^{30}$ Lunch

Session Chair: Alessandro Fedrizzi

$15^{00} - 16^{00}$ Alain Joye (Universite Grenoble), Transport and Spectral Properties of Random Quantum Walks

$16^{00} - 16^{30}$ Iwao Sato (Oyama National College of Technology), A note on the discrete-time evolutions of quantum walk on a graph

$16^{30} - 17^{00}$ Tea/Coffee

$17^{00} - 17^{30}$ Maria Schuld (University of KwaZulu-Natal), Using quantum walks for quantum associative memory

$19^{00} - 21^{00}$ Conference Dinner

Thursday, 27 November

Session Chair: Alain Joye

$9^{00} - 10^{00}$ Oliver Mülken (University of Freiburg), Quantum Walks on Complex Networks

$10^{00} - 10^{30}$ Alessio Celi (ICFO - The Institute of Photonic Sciences), Synthetic Quantum Walk from (synthetic) edge states with ultracold atoms in optical lattices

$10^{30} - 11^{00}$ Tea/Coffee

$11^{30} - 12^{00}$ Chandrashekar Madaiah (Okinawa Institute of Science and Technology), Quantum percolation on a two-dimensional lattice with random disconnection using quantum walks

$12^{00} - 12^{30}$ Jaroslav Novotný (FNSPE CTU, Prague), Coined quantum walk on dynamically percolated finite grids

$12^{30} - 14^{30}$ Lunch

Session Chair: Oliver Mülken

$15^{00} - 16^{00}$	Jacques Carolan (University of Bristol), Verifying the unverifiable: certifying quantum complexity in linear optical experiments
$16^{00} - 16^{30}$	Andrea Alberti (Institut für Angewandte Physik, University of Bonn), Decoherence Models for Discrete-Time Quantum Walks and their Application to Neutral Atom Experiments
$16^{30} - 17^{00}$	Tea/Coffee
$17^{00} - 17^{30}$	Martin Stefanak (Czech Technical University in Prague), Controlling quantum walks with coin eigenstates.
$17^{30} - 18^{00}$	Iva Bezdekova (Czech Technical University in Prague), Localization and limit distribution for quantum walks
$19^{00} - 21^{00}$	Dinner

Friday, 28 November

Session Chair: Ilya Sinayskiy

$9^{00} - 10^{00}$	Tatsuya Tate (Tohoku University), Localization, eigenvalues and absolute continuity for periodic unitary transition operators
$10^{00} - 10^{30}$	Yutaka Shikano (Institute for Molecular Science), Massless Dirac equation from discrete-time quantum walk
$10^{30} - 11^{00}$	Tea/Coffee
$11^{00} - 11^{30}$	Stefan Boettcher (Emory University), Renormalisation of Quantum Walks
$11^{30} - 12^{00}$	Thomas Konrad (University of KwaZulu-Natal), A scalable implementation of quantum walks using classical light
$12^{00} - 12^{30}$	Concluding remarks
$12^{30} - 14^{30}$	Lunch
$14^{30} - ...$	Departure/Excursion

List of Abstracts

Monday, 24 November

Classical Noise in Quantum Markov chains
Stephane Attal

We consider quantum Markov chains described by repeated quantum interactions with some quantum environment. Among those actions of a quantum environment we characterise those which induce a classical noise on the small system. We show that these classes of interactions are described by particular random variables: the obtuse random variables. These random variables are associated to nice algebraical properties such as the diagonalizability of 3-tensors. The diagonalization in particular drives the continuous-time limit behaviour of these noises.

Asymptotic properties of the Dirac quantum cellular automaton
Armando Perez

It is well known that the Discrete Time Quantum Walk (DTQW) contains some properties that are related to the Dirac equation. On the other hand, the Dirac Quantum Cellular Automaton (DQCA) studied in (arXiv:1212.2839) and in (PRA 88, 032301) contains similar features, and a time evolution that resembles the DTQW. We investigate the asymptotic properties of the DQCA in connection with the DTQW.

Discrete Lorentz transformations for Quantum Walks and Quantum Cellular Automata
Pablo Arrighi, Stefano Facchini and Marcelo Forets

We formalize a notion of discrete Lorentz transforms for Quantum Walks (QW) and Quantum Cellular Automata (QCA), in $(1 + 1)$-dimensional discrete spacetime. The theory admits a diagrammatic representation in terms of a few local, circuit equivalence rules. Within this framework, we show the first-order-only covariance of the Dirac QW. We then introduce the Clock QW and the Clock QCA, and prove that they are exactly discrete Lorentz covariant.

Tuesday, 25 November

The Open Quantum Brownian Motion: Fluctuation and (possible) Geometrisation

Denis Bernard

Using quantum parallelism on random walks as original seed, I shall introduce the Open Quantum Brownian Motion model. It describes the behavior of quantum walkers interacting with series of probes which serve as quantum coins and may be viewed as the scaling limit of Open Quantum Random Walks. To deal with the fluctuating aspects, I shall describe a transition in its statistical behavior from a diffusive to a ballistic behavior at intermediate scales. To deal with possible geometrical aspects, I shall discuss a possible route to extend its definition to two dimensions while preserving the known conformal invariance of its classical ancestor.

An example of the microscopic derivation of Open Quantum Brownian Motion

Ilya Sinayskiy and Francesco Petruccione

The microscopic derivation of Open Quantum Brownian Motion (OQBM) is presented. The quantum master equation for OQBM is derived for a weakly driven system interacting with the decoherent environment. Examples of the dynamics for initial Gaussian and non-Gaussian distributions are presented. Both examples demonstrate convergence of the OQBM dynamics to Gaussian distributions.

Open Quantum Walks

Ilya Sinayskiy and Francesco Petruccione

Over the last few years dynamical properties and limit distributions of Open Quantum Walks (OQWs), quantum walks driven by dissipation, have been intensely studied [S. Attal et. al. J. Stat. Phys. 147, Issue 4, 832 (2012)]. For some particular cases of OQWs central limit theorems have been proven [S. Attal, N. Guillotin, C. Sabot, "Central Limit Theorems for Open Quantum Random Walks," Annales Henri Poincare]. It have been shown that OQWs can be used to perform dissipative quantum computation and quantum state engineering [I. Sinayskiy and F. Petruccine, QIP (2012) 11:1301]. However, only recently the connection between the rich dynamical behaviour of OQWs and the corresponding microscopic system-environment models has been established. The microscopic derivation of an OQW as a reduced system dynamics on a 2-nodes graph [I. Sinayskiy, F. Petruccione, Open Syst. Inf. Dyn. 20, 1340007 (2013)] and its generalisation to arbitrary graphs allow to explain the dependance of the dynamical behaviour of the OQW on the temperature and coupling to the environment. Physical realisations of OQWs in quantum optical setups will be also presented [I. Sinayskiy and F. Petruccione, IJQI, 12, 2(2014)1461010].

Open Quantum Self-Avoiding Walks

Hazmatally Goolam Hossen, Ilya Sinayskiy and Francesco Petruccione

Our aim is to design an open quantum self-avoiding walk (SAW). Firstly, we examine the open quantum walk introduced by Attal et al (Attal et al, 2012 J Stat Phys 147:832852). Basically it is the quantum version of a classical Markov chain but whereby the states are represented by density matrices whose traces give the probability. Afterwards, we review unitary quantum SAWs analysed by Machida et al on subspaces of the complete Hilbert space (Machida et al, arXiv: quant-ph /1307.6288v2). In the coin space, it is called the non-repeating walk and in the position space, the non-reversal walk. Proctor et al investigate these two alternatives to completely self-avoiding walks and showed that they are both interconnected by a permutation of the coin operator (Proctor et al, 2014 Phys. Rev. A 89, 042332). In a different setup by Camilleri et al, the self-avoidance of the quantum walk is related to the strength of the memory recording and back-action (Camilleri et al, doi:10.1038/srep04791). After studying both the unitary and non-unitary quantum walks, we formulate the one-dimensional open quantum non-reversal walk whereby the particle in each step can either move to a previously unoccupied position or stay where it was. Of course, we obtain a unidirectional walk. We are concerned with its final position after a fixed number of steps. Thus, we run the simulation enough times so as to obtain a probability distribution of these final positions which we then compare with that of an ordinary open quantum walk. Our next step is to explore the possibility of doing the same for a two-dimensional open quantum SAW.

Hamiltonian simulation with nearly optimal dependence on all parameters

Dominic Berry, Andrew Childs and Robin Kothari

We present an algorithm for sparse Hamiltonian simulation that has optimal dependence on all parameters of interest (up to log factors). Previous algorithms had optimal or near-optimal scaling in some parameters at the cost of poor scaling in others. Hamiltonian simulation via a quantum walk has optimal dependence on the sparsity d at the expense of poor scaling in the allowed error epsilon. In contrast, an approach based on compressed product formulas provides optimal scaling in epsilon at the expense of poor scaling in d. Here we combine the two approaches, achieving the best features of both. By implementing a linear combination of quantum walk steps with coefficients given by Bessel functions, our algorithm achieves near-linear scaling in $\tau := td|H|_{max}$ and sublogarithmic scaling in $1/\epsilon$. Our dependence on epsilon is optimal, and we prove a new lower bound showing that no algorithm can have sublinear dependence on tau.

Noncommutative polynomial optimization and simulations with quantum resources

Miguel Navascues, Tamás Vértesi and Andreas Winter

Noncommutative polynomial optimization (NPO) studies the problem of minimizing polynomials of noncommuting variables subject to non-trivial polynomial constraints. Such variables must be understood as operators acting on a not-a-priori-defined Hilbert space, whose dimension can be fixed by the user. Dimension-constrained NPO problems appear naturally in quantum communication complexity, where the goal is to determine the minimum number of communicated qubits needed to simulate the computation of a bivariate function. Here I will introduce a practical hierarchy of SDP relaxations to solve NPO problems with dimension constraints. I will discuss the efficiency of the new hierarchy by applying it to solve a number of simulation problems which were previously thought to be intractable in a normal desktop.

Digital quantum simulation of single-qubit open quantum systems

Ryan Sweke, Ilya Sinayskiy and Francesco Petruccione

In the last two decades a large amount of effort has been focused on developing methods for the digital quantum simulation of closed quantum systems, undergoing Hamiltonian generated unitary evolution. However it is only more recently that methods have been suggested for the digital quantum simulation of open quantum systems, crucial for enhancing our understanding of non-equilibrium dynamics and thermalization in a wide range of systems. In this talk I will review current approaches to the digital quantum simulation of open quantum systems as well as present a recent algorithm for the simulation of single-qubit open quantum systems. Inspired by developments in Hamiltonian simulation a decomposition and recombination technique is utilized for this algorithm, which allows for the exploitation of recently developed methods for the approximation of arbitrary single-qubit channels. In particular, as a result of these methods the algorithm requires only a single ancilla qubit, the minimal possible dilation for a non unitary single-qubit quantum channel.

Lattice shaking for topological effects in cold atomic quantum simulators.

Anna Przysiezna, Omjyoti Dutta and Jakub Zakrzewski

Topologically protected states of matter have amassed great attention due to their novel properties such as robust transport on the boundary (in 2D and 3D) and localized edge states (in 1D). In the field of atomic quantum simulators there are ongoing studies aiming to create non-trivial lattices supporting topological states by optical means. We present a different approach to realize such structures based on self organization of atoms. We consider attractive ultracold fermions trapped in optical lattices of different geometries and show that nontrivial structures can emerge from the combined effect of attractive interactions and lattice shaking. We consider two different geometries: In a one dimensional system, we find a regime of parameters where atoms self-organize into a dimerized structure and paradigmatic dimer physics described by the Rice-Mele model is realized. Our system exhibits zero-energy edge states and has a nontrivial Zak phase which certifies the topological nature. In the second system under considerations - shaken two dimensional triangular lattice - we find that the above mentioned effects can result in an emergent Dice lattice along with controllable staggered magnetic flux and synthetic non-Abelian fields. Moreover, by tuning the staggered flux, one can enter the regime of Quantum Anomalous Hall effect. We complement theoretical deriviatons discussing experimental methods that may be used in order to realize the systems.

Wednesday, 26 November

Observation of environmentally-assisted quantum transport in a photonic emulator
Alessandro Fedrizzi

The discovery of quantum coherence in photosynthetic processes has sparked enormous interest in the exact role of quantum effects in biology. Some of the biggest open questions are how robust the coherent dynamics in photosynthesis is; whether it really assists energy transport; and whether it is optimised by natural selection or just a random by-product. These questions are difficult to address experimentally, since the structure of a biological complex cannot usually be modified. In this talk I willreport progress in photonic emulation of Hamiltonians occurring in natural processes in 3D direct-write waveguide arrays. Our system allows control over the system dynamics, the degree of coherence, and the environment, with the goal of understanding how these factors interact in modifying quantum transport. In particular, we study the coherent evolution of excitons undergoing the Hamiltonian governing photosynthesis in the purple bacterium rhodobacter sphaeroides, and report results on decoherence-assisted enhancement of quantum transport in a simple 4-site system coupled to a photonic bath.

Quantum walks on infinite trees
Etsuo Segawa

We consider discrete-time quantum walks on infinite trees. We obtain limit theorems implying the co-existence of linear spreading and localization in the subspace mapped onto a half integers. We also discuss a hyperbolicity of the tree structure reflected by this quantum walk.

Bulk-edge correspondence in one-dimensional quantum walks
Christopher Cedzich

In this talk we will explore the notion of bulk-edge correspondence in one-dimensional quantum walks, a phenomenon well-known from solid-state physics. To this end we examine the notion of topological invariants and nail down a meaningful definition. Examples introduced by Kitagawa et al. will be studied with the necessary level of mathematical rigor.

Quantum Walks in Electromagnetic Fields

Christoph Stahl, Christopher Cedzich, Albert Werner, Reinhard F. Werner

Building on recent results describing the electrification of 1D quantum walks (Cedzich et al. Phys. Rev. Lett., 111:160601), we add magnetic fields to our description of dis- crete time quantum walks by introducing magnetized translation operators obeying commutation relations analogous to those in continuous time systems. In the case of homogeneous magnetic fields, numerical signatures of different spreading behaviors are found depending on the rationality of the fields. We use representation theoretical methods combined with a description in terms of the rotation algebra to further classify magnetic quantum walks. This allows us to computationally reproduce the Hofstadter Butterfly (Hofstadter Phys. Rev. B, 14:2239) as a self-similar representation of the combined spectrum of magnetic quantum walks.

Transport and Spectral Properties of Random Quantum Walks

Alain Joye

We review the construction of unitary quantum walks on infinite graphs in presence of static or time-dependent disorder. Then we describe several recent mathematical results about the transport and spectral properties of the corresponding random unitary operators such walks give rise to.

A note on the discrete-time evolutions of quantum walk on a graph

Norio Konno, Yusuke Higuchi, Iwao Sato and Etsuo Segawa

Recently many researchers in various fields pay attention to the quantum walk on graphs. The Grover evolution matrix of a graph is efficient for the graph isomorphism problem, and various approach are done in the graph isomorphism problem. The spectrum of the Grover evolution matrix of a graph G is given from that of the transition operator of a simple random walk on G. Furthermore, the Szegedy evolution matrix of a quantum walk on a graph G is given from that of the transition operator of a random walk on G. One of our main purposes is to generalize the above facts on the Grover evolution matrix and the Szegedy evolution matrix. We present a generalized evolution matrix of a graph and compute its characteristic polynomial.

Using quantum walks for quantum associative memory

Maria Schuld, Ilya Sinayskiy and Francesco Petruccione

Artificial neural networks are mathematical models that simulate essential computational properties of biological neural networks. In particular, Hopfield networks converge to memorised states that are stored in the model's parameters, realising a so called associative memory. The dynamics of neural networks can be understood as the evolution of a binary string encoding the state of each

neuron in the network. Various attempts of finding a quantum neural network model use quantum theory to reproduce and improve the striking properties of neural networks. As part of this effort, we use Stochastic Quantum Walks on graphs with nodes that represent the states of a neural network in order to explore associative memory based on quantum evolution. We show that the quantum nature of the dynamics indeed lead to a moderate speed-up in finding the closest memorised state.

Thursday, 27 November

Quantum Walks on Complex Networks
Oliver Mülken

I will present several aspects of quantum transport processes on complex networks. After introducing the basic concepts for modelling quantum dynamical processes over complex structures, e.g., (macro-)molecules or ultra-cold gases, which can be modelled by networks, I will present examples where the quantum dynamics shows interesting features such as localisation which can be overcome by introducing disorder into the system. The quantum dynamics will be compared to the classical diffusive dynamics over the same classes of networks. I will further discuss results on the temporal entropy growth for the quantum-classical crossover by employing methods from the theory of open quantum systems.

Synthetic Quantum Walk from (synthetic) edge states with ultracold atoms in optical lattices
Alessio Celi

In this talk, I will discuss the possibility of simulating a quantum walk using edge states. This opens the possibility of realizing in running experiments a topological insulator, which governs the dynamics of the quantum walk, by dressing the dynamics of another topological insulator, namely an integer quantum Hall.

Quantum percolation on a two-dimensional lattice with random disconnection using quantum walks
Chandrashekar Madaiah and Thomas Busch

While quantum percolation is known to have certain similarities with classical percolation, the quantum case has additional complexity due to the possibility of Anderson localisation. In this talk we present a model to simulate and determined the fraction of connected edges required (the transition point) in a lattice for a two-state particle to percolate with finite (non-zero) probability, using a directed quantum walk dynamics. Three fundamental lattice geometries (finite square lattice, honeycomb lattice, and nanotube structure) are examined and we show that the transition point each time tends towards unity for increasing lattice sizes. To support the numerical results we also use a continuum approximation to analytically derive the expression for the percolation probability for the case of the square lattice and show that it agrees with the numerically obtained results for the discrete case. Beyond the fundamental interest to understand the dynamics of a two-state particle on a lattice (network) with disconnected vertices, this study lays the foundations to shed light on the transport dynamics in various quantum condensed matter systems and the construction of quantum information processing and communication protocols in imperfect situations. We will also present our study of photons percolation in an imperfect array of beam-splitters, where the dynamics is determined by the interplay between percolation, backscattering and transient localisation. From both these studies one can conclude that even with a directed nature of the evolution, a very high percentage (close to 100%) of well connected lattice sites is required to obtain a non-zero percolation probability even on lattices of finite size. These exact values for the transition points on finite lattices however dependent on the lattice type. These studies corroborate the effective use of quantum walks to simulate quantum percolation.

Coined quantum walk on dynamically percolated finite grids

Jaroslav Novotný, Balint Kollar, Tamas Kiss and Igor Jex

Quantum walks on graphs constitute an efficient tool of quantum information capable to capture different features of quantum evolution. Discrete quantum walk dynamics results from repeated applications of a basic step, which comprises evolution of internal degrees of freedom (defined by the coin operator) followed by the conditional shift in the position space (governed by the step operator). The richness of quantum walk dynamics arises from the spectrum of graph structures as well as from variety of coin operators. Towards a more realistic scenario one naturally encounters arising complexity of designed models. We study coined quantum walk on finite grids which suffers from eventually broken links among vertices. In each single step the underlying graph is randomly alternated - quantum walk on dynamically percolated grid. As solely the probability of an actual graph at a given step is known the total dynamics is a highly complex incoherent mixture of unitary evolutions corresponding to different graphs. Based on the theory of random unitary operations we introduce a novel method which provides analytic closed solutions of asymptotic dynamics for any initial state. In particular, we present solution of quantum walk on percolated 1D and 2D finite grids. Special choices of coin operators reveal interesting effects like asymptotic position inhomogeneity, special directional symmetry breaking in position distribution or an existence of so-called edge states. Moreover, we show how imperfect connections of percolated latices increase an efficiency of a transport of the walker to a sink (a special vertex, in which the walker is forced to leave the grid permanently).

Verifying the unverifiable: certifying quantum complexity in linear optical experiments

Jacques Carolan

The first quantum technologies to solve computational problems that are beyond the capabilities of classical computers are likely to tackle bespoke problems suited to their own physical characteristics. Evidence implies that boson sampling, detecting ensembles of photons after propagation through a linear optical circuit, is equivalent to sampling from a probability distribution that is intractable to classical simulation. However, the complexity of this type of problem means its solution is unverifiable and the task of establishing correct operation becomes one of gathering sufficiently convincing circumstantial evidence. Here, we develop scalable methods to experimentally establish correct operation for this class of sampling algorithm, which we implement with two different types of optical circuits for 3, 4, and 5 photons, on Hilbert spaces of up to 50,000 dimensions. We show that by exploiting a predictable structure in the probability distribution of photonic quantum walks, bosonic clouding, efficient unitary discrimination can be achieved. Our proposal thus uses the predictable correlations in highly structured unitaries as a verification tool for the formally unverifiable boson sampler.

Decoherence Models for Discrete-Time Quantum Walks and their Application to Neutral Atom Experiments

Andrea Alberti, Wolfgang Alt, Reinhard Werner and Dieter Meschede

Understanding decoherence mechanisms is essential to develop future quantum technologies like machines capable of realizing complex tasks through a series of coherent quantum operations. The discrete-time quantum walk on the line is a prime example of such a machine. Within a phenomenological approach, we study two classes of decoherence mechanisms, which either affect the external or internal degree of freedom of the quantum walk (arXiv:1409.6145 [quant-ph] (2014)). We find that our decoherence model is able to correctly reproduce our experimental data up to about 100 steps, which are obtained in a neutral atom experiment. In addition to the phenomenological approach, we also present an overview of the most important physical decoherence mechanisms in optical lattice systems. With respect to previous studies, which exclusively hinge on the spatial probability distribution, our analysis provides new physical insight by investigating the evolution of spatial and spin coherences. We introduce, for instance, the concept of coherence length to precisely quantify the range of spatial coherences. Objective criteria that assess the quality of the quantum walk beyond the simple analysis of the spreading speed are essential to advance the state of the art in future experiments.

Controlling quantum walks with coin eigenstates

Martin Stefanak, Iva Bezdekova and Igor Jex

We show that the control of the Hadamard walk and the three-state Grover walk on a line is made particularly transparent when the initial state is expressed in terms of the eigenstates of the coin operator. In particular, we show that the limit distributions acquire a much simpler form when expressed in this basis. This allows us to obtain a much deeper understanding of the role of the initial coin state on the dynamics of quantum walks. In particular, we find that the eigenvectors of the coin leads to very unusual probability distributions.

Localization and limit distribution for quantum walks

Iva Bezdekova, Martin Stefanak and Igor Jex

The localization effect of the quantum walk is characterized by a non-vanishing probability of the particle to stay at the origin. This interesting property is sensitive to the dimensionality of the lattice and the choice of the coin operator. As an example, the effect was found in the three-state Grover walk on a line. In the dimension three and higher, there exist two simple deformations of the Grover coin that preserve localization, let us call it eigenvalue and eigenvector family. Moreover, the results for the three-state walk on a line can be generalized in order to provide all coins leading to the localizing walk. The case of eigenvalue and eigenvector family can be analyzed in more details, from the viewpoint of the limit distribution. Appropriate choice of the basis considerably simplifies the description of the walk. we show that the proper basis here is formed by the eigenvectors of the coin operator.

Friday, 28 November

Localization, eigenvalues and absolute continuity for periodic unitary transition operators

Tatsuya Tate

One of typical properties of asymptotic behavior of transition probabilities of quantum walks are their localization. The localization happens even for quantum walks with constant coin matrix. In the talk, the mathematical structures of localization phenomenon of the transition probabilities for periodic unitary transition operators on a Hilbert space of square summable functions on an integer lattice with values in a complex vector space, which is a generalization of the discrete-time quantum walks with constant coin matrices, will be discussed. As is expected in the theory of quantum walks, the spectral structure of periodic unitary transition operators, such as eigenvalues and absolute continuity of the continuous spectrum, is closely related to the localization phenomenon. Some theorems concerning the relationships among eigenvalues, absolute continuity of continuous spectrum and localization phenomenon for the periodic unitary transition operators will be given.

Massless Dirac equation from discrete-time quantum walk

Giuseppe Di Molfetta, Lauchlan Honter, Ben Luo, Tatsuaki Wada and Yutaka Shikano

Two different type Fibonacci discrete-time quantum walks are studied analytically. One model is the Fibonacci coin sequence with the generalized Hadmard coin and has the six step periodic dynamics. The other model is assumed the three or six step periodic dynamics with the Fibonacci sequence, which has already discussed. We analytically show that these models have the ballistic transportation properties and these continuous limits are identical to the massless Dirac equation with the coin basis change.

Renormalization of Quantum Walks

Stefan Boettcher, Stefan Falkner and Renato Portugal

I will describe the renormalization group method (RG) from statistical physics as applied to master equations with a unitary propagator. It allows to determine many asymptotic properties of quantum walks, although I will focus here on the walk dimension d_w, which describes the similarity solution, $\rho(x,t) \sim f(|x|^{d_w}/t)$, for the probability density function ρ. We can calculate d_w to arbitrary accuracy for a number of networks, such as the dual Sierpinksi gasket, small-world Hanoi networks, or Migdal-Kadanoff lattices, which we have verified with direct simulations. However, due to unitarity, the asymptotic solution of the RG equations as well as procedures to implement RG approximately for arbitrary networks remain elusive. Yet, based on the exact RG for those fractal networks, we can conjecture a few general conclusions, for instance, that d_w for a discrete-time quantum walk is always half of that for the random walk on the same r-regular network, when driven with the Grover coin.

A scalable implementation of quantum walks using classical light

Thomas Konrad and Sandeep Goyal

We study the notion of classical entanglement at the example of a quantum walk implemented by means of the orbital angular momentum and the polarization degree of freedom of a classical light beam. The scheme makes use of a ring interferometer containing a quarter wave plate and a q plate. While the polarization serves here as the coin to determine the direction of the walk in each step, the orbital angular momentum represents the position of the walker which increases or decreases depending on the polarization due to the action of the q plate. The coupling of both optical degrees of freedom leads to optical states which are reminiscent of entangled states of quantum systems and are called classically entangled. The generation of classically entangled states is a prerequisite to mimic coined quantum walks.